APPENDIX 8.A — EXAMPLE PROBLEMS

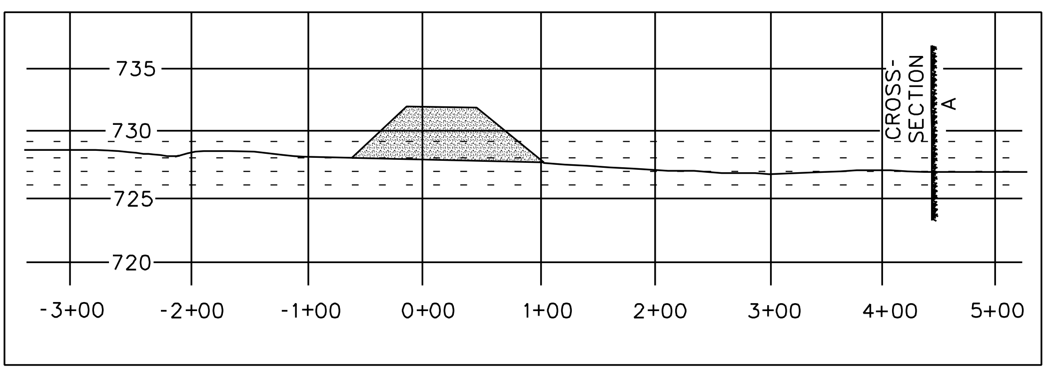
Example 8.A-1

Given: The 25-yr and 100-yr peak discharges are Q_{25} = 175 ft ^{3}/s and Q_{100} = 219 ft^3/s. Cross section information is given in the following table of surveyed data points for a typical cross section.

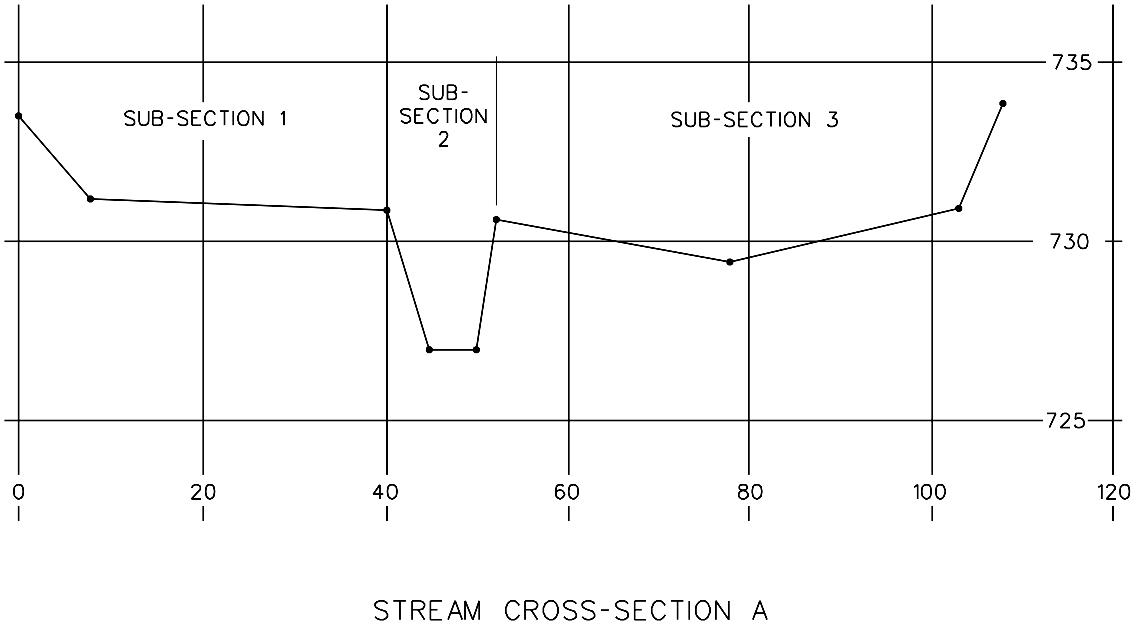
Table of Cross Section Data (Section "A") – Sta. 4+45

Distance	Elevation	n-Value
0	733.5	0.06
8	731.2	0.06
40	730.8	0.035
45	727.0	0.035
50	727.0	0.035
53	730.6	0.05
78	729.4	0.05
103	731.0	0.05
108	734.0	0.05

Subsection 1 consists of an overbank area with light brush and trees. Subsection 2 is in the main channel of this stream and comprises a clean, straight stream with a few weeds and rocks. Subsection 3 is in the right floodplain and includes some scattered brush with considerable weediness.



Find: Use the single-section method to develop a stage-discharge curve for the channel cross section at Station 4+45, which is located downstream from a highway culvert. Determine the tailwater elevation at the outlet of the culvert (assume a channel Station of 1+00 for this location) for the 25-yr and 100-yr floods.



Stream Cross Section "A"

Solution: The slope of the stream can be determined by examining the reach from stream Station -3+00 to the typical section at Station 4+45. The flow line differential for this reach is 2.00 ft (in 745 ft of stream reach). Therefore, the slope (S) is 0.0027 ft/ft.

Figure 8.A-1 can be used to assist in the development of a stage-discharge curve for this typical section. Assuming water surface elevations beginning at 728.00, calculate pairs of water surface elevation/discharge for plotting on a stage-discharge curve. Illustrative calculations in which arbitrary increments of water surface elevation of 1.0 ft were used are shown in Figure 8.A-1. A plotted stage-discharge curve is shown in Figure 8.A-2. The water elevation for Q_{25} (175 ft^3/s) is 731.6 and, for Q_{100} (219 ft^3/s), 731.8.

Because the calculation section for the stream is downstream of the culvert site, it will be necessary to project the water surface elevation as determined from the typical section at stream Station 4+45 to represent the tailwater elevation at stream Station 1+00. Therefore, the projected tailwater levels are calculated as follows:

$$TW_{25} = 731.6 + (445 - 100)(0.0027) = 732.53 \text{ ft}$$

$$TW_{100} = 731.8 + 0.93 = 732.73 \text{ ft}$$

Elevation = 728.0 ft	Slope = 0.0027						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)		6.0					6.0
Wetted Perimeter (ft)		7.9					
Hydraulic Radius (ft)		0.76					
$R^{2/3}$		0.83					
n	0.060	0.035	0.050				
ΔQ (ft^3/s)		11.0					11.0
Subsection Vel. (ft/s)		2.3					2.3

Elevation = 729.0 ft	Slope = 0.0027						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)		14.3					14.3
Wetted Perimeter (ft)		10.9					
Hydraulic Radius (ft)		1.31					
$R^{2/3}$		1.20					
n	0.060	0.035	0.050				
ΔQ (ft^3/s)		37.9					37.9
Subsection Vel. (ft/s)		2.8					2.8

Elevation = 730.0 ft	Slope = 0.0027						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)		24.6	6.6				31.2
Wetted Perimeter (ft)		14.9	21.9				
Hydraulic Radius (ft)		1.65	0.30				
$R^{2/3}$		1.40	0.45				
n	0.060	0.035	0.050				
ΔQ (ft^3/s)		76.0	4.6				80.6
Subsection Vel. (ft/s)		3.1	0.7				2.6

FIGURE 8.A-1 — Channel Computation Form

Elevation = 731.0 ft	Slope = 0.0027						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)	1.6	37.1	45.0				83.7
Wetted Perimeter (ft)	16.0	16.0	50.1				
Hydraulic Radius (ft)	0.1	2.25	0.90				
$R^{2/3}$	0.22	1.72	0.93				
n	0.060	0.035	0.050				
ΔQ (ft^3/s)	0.5	140.8	64.6				205.9
Subsection Vel. (ft/s)	0.3	3.8	1.4				2.5

Elevation = 732.0 ft	Slope = 0.0027						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)	36.5	49.0	95.8				181.3
Wetted Perimeter (ft)	34.8	16.0	52.1				
Hydraulic Radius (ft)	1.05	3.06	1.84				
$R^{2/3}$	1.03	2.11	1.50				
n	0.060	0.035	0.050				
ΔQ (ft^3/s)	48.4	228.1	221.9				498.4
Subsection Vel. (ft/s)	1.3	4.7	2.3				2.7

Elevation =	Slope =						
Subsection ID	I	II	III	IV	V	VI	Totals/Average
Area (ft^2)							
Wetted Perimeter (ft)							
Hydraulic Radius (ft)							
$R^{2/3}$							
N							
ΔQ (ft^3/s)							
Subsection Vel. (ft/s)							

FIGURE 8.A-1 — Channel Computation Form
(Continued)

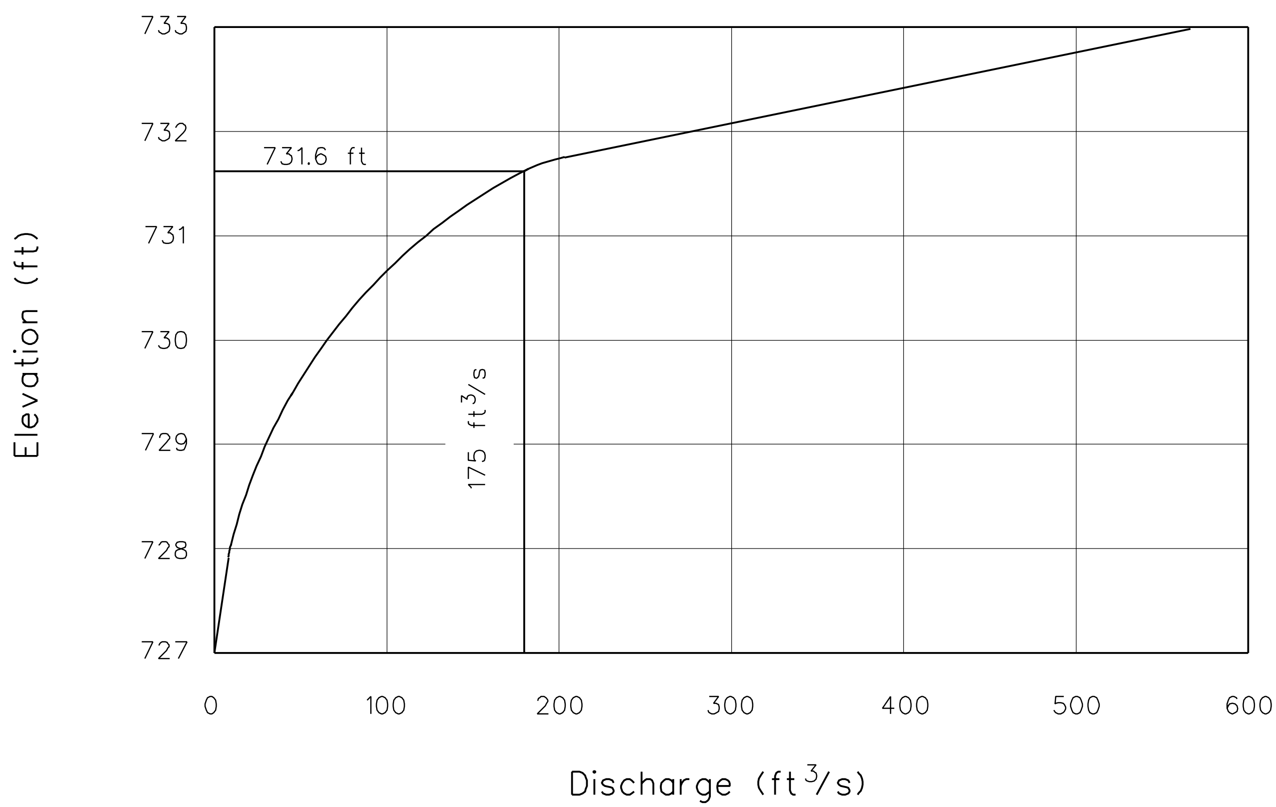


FIGURE 8.A-2 — Example Stage-Discharge Curve

Example 8.A-2

The step-backwater procedure is illustrated in the following Example.

Four cross sections along a reach are shown in Figures 8.A-3 through 8.A-6. Each cross section is separated by 152.4 m and is subdivided according to geometry and roughness. The calculations shown in Table 8.A-1 represent one set of water-surface calculations. An explanation of Table 8.A-1 is in Section 8.5.4.3.

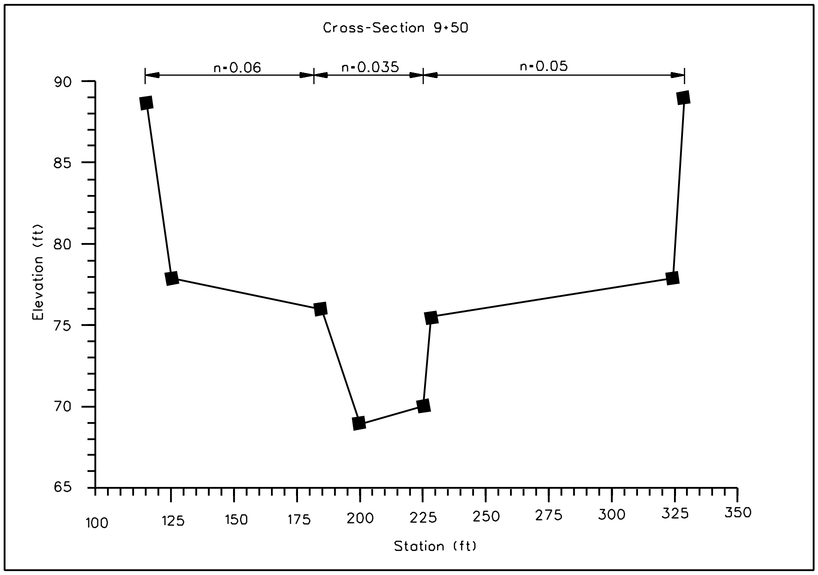


FIGURE 8.A-3 — Cross Section at Station 9.79 (farthest upstream)

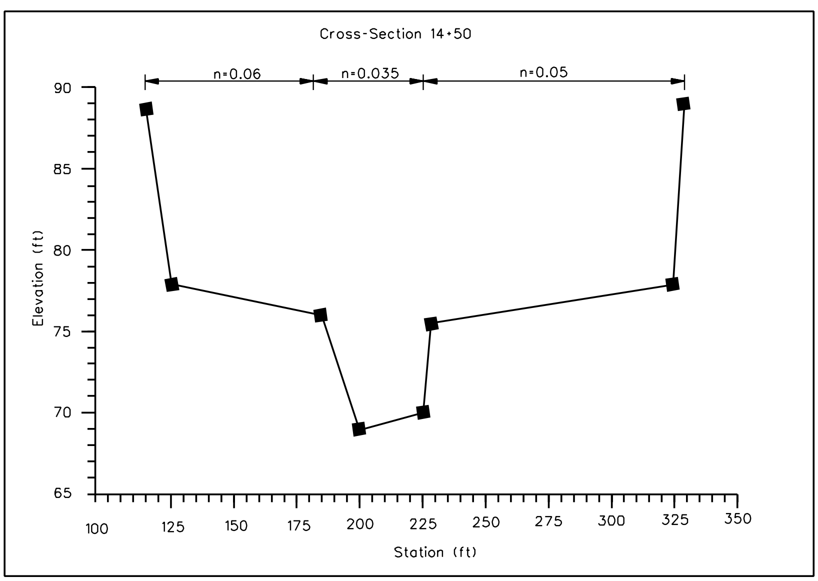


FIGURE 8.A-4 — Cross Section at Station 9.70

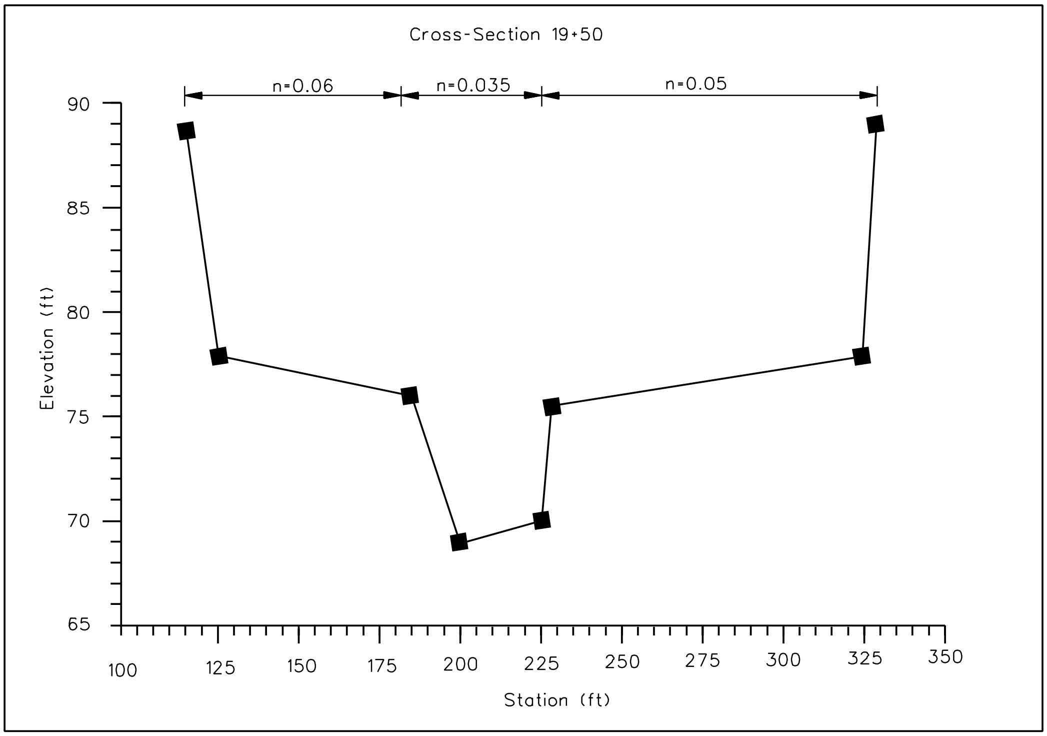


FIGURE 8.A-5 — Cross-Section at Station 19 + 50

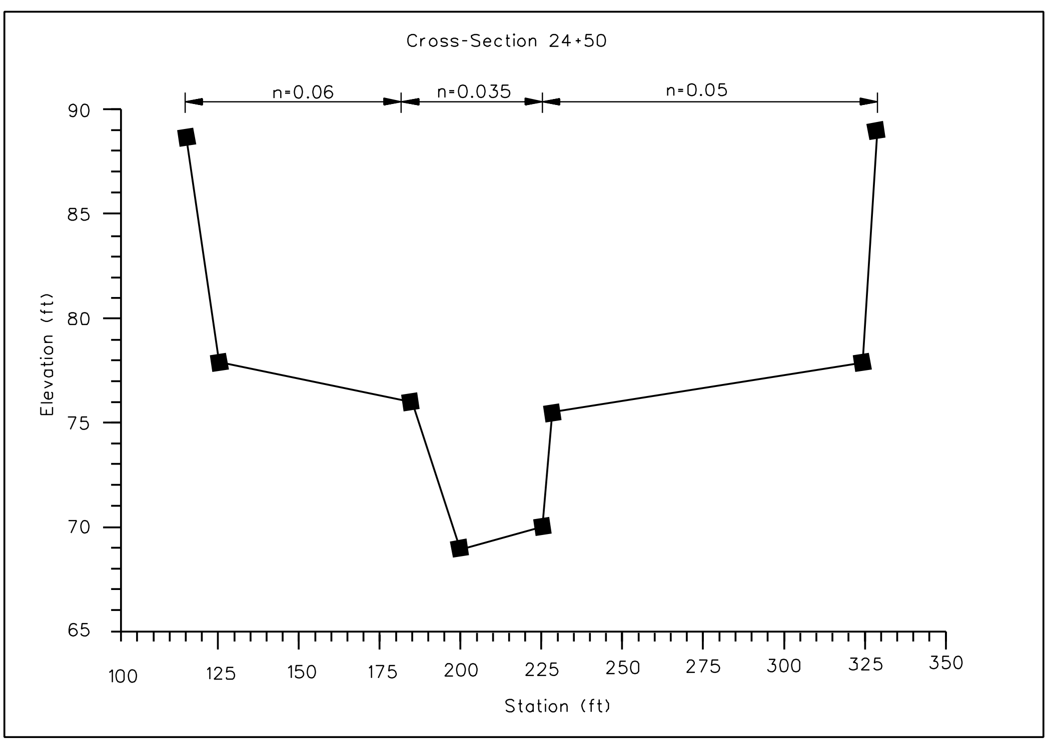


FIGURE 8.A-6 — Cross Section at Station 24 + 50 (farthest downstream)

TABLE 8.A-1

Cross Section No.	Water Surface Elevation Assumed	Water Surface Elevation Computed	Area	R	$R^{2/3}$	n	K	$\bar{K}_t$	$1000S_f$	L	h_f	$\Sigma(K^3/A^2)$	α	V	$\alpha V^2/2g$	$\Delta(\alpha V^2/2g)$	h_o	Δ Water Surface Elevation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
9+50		79.30																
			191	2.23	1.71	0.060	8089					14,508,342						
			506	8.20	4.07	0.035	87437					2,610,866,774						
			146	1.28	1.18	0.050	5120					6,296,572						
			843				100646	100646	0.40	0	0	2,631,671,688	1.83	2.4	0.16	0.00	0.000	0.00
14+50	79.50	79.50																
			192	2.23	1.71	0.060	8131					14,582,414						
			507	8.20	4.07	0.035	87610					2,616,044,470						
			147	1.28	1.18	0.050	5155					6,339,446						
			846				100896	100771	0.40	500	0.20	2,636,966,330	1.84	2.4	0.16	0.00	0.000	0.20
19+50	80.22	79.73																
			227	2.66	1.92	0.060	10794					24,405,939						
			531	8.60	4.20	0.035	94688					3,010,896,233						
			196	1.71	1.43	0.050	8330					15,046,063						
			954				113812	107354	0.35	500	0.18	3,050,348,235	2.24	2.1	0.15	0.10	0.010	0.21
	79.66	79.66																
			182	2.17	1.68	0.060	7573					13,111,763						
			501	8.10	4.04	0.035	85935					2,528,336,022						
			136	1.18	1.12	0.050	4527					5,015,955						
			818				98035	105923	0.36	500	0.18	2,546,463,740	1.43	2.5	0.14	0.01	0.005	0.18
24+50	79.89	79.86																
			130	1.87	1.52	0.060	4894					7,086,616						
			403	7.61	3.87	0.035	66217					1,787,715,520						
			84	0.95	0.97	0.050	2422					2,013,557						
			617				73533	85784	0.55	500	0.28	1,796,815,693	1.08	3.3	0.18	-0.04	0.014	0.20

Q=2013 ft^3/s

Example 8.A-3

Given: A roadside drainage channel is trapezoidal with a bottom width of 4 ft and 1V:3H side slopes. The bed slope is 0.005 ft/ft and the design flow rate is 21 ft^3/s.

Find: Calculate the required diameter (D_{50}) of a gravel riprap that is to be used as a permanent channel lining and the design depth of flow.

Solution: The solution follows the procedure outlined in HEC 15 *(11)*, which is based on the tractive force method:

(1) Choose a rounded gravel with D_{50} = 1 in

Then $\tau_p = 0.40$ lb/ft^2 (Table 8-4)

(2) Estimate n = 0.033 from Table 8-5 for y = 0.5 – 2.0 ft

(3) Calculate y from Manning's equation (Figure 8-4):

$[(1.486nQ)/(b^{8/3})(S^{1/2})] = [1.486(0.033)(21)]/[(4.0^{8/3})(0.005^{1/2})] = 0.36$

Then, from Figure 8-4 with Z = 3: y/b = 0.34 and y = 1.36 ft

(4) Check n value:

$R = [y(b + 3d)]/[b + (2y)(10^{1/2})] = 0.67$ ft and $R/k_s = R/D_{50} = 8.0$

and from Figure 8-10, n = 0.034 . 0.033 OK

(5) Calculate maximum bed shear stress, τ_d:

$\tau_d = 62.4yS = (62.4)(1.36)(0.005) = 0.42$ lb/ft^2

Now, because $\tau_d \approx \tau_p$, accept D_{50} of approximately 1-in. Otherwise, repeat with another riprap diameter.

(6) Side slopes will be stable because side slope is not steeper than 1V:3H. If side slopes are steeper than 1V:3H or if channel slope is steep, consult HEC 15 for additional computations.

Example 8.A-4 (From HEC 15 *(11)*)

Given: A median ditch is lined with a good stand of Kentucky bluegrass (approximately 8 in in height). The ditch is trapezoidal with a bottom width of 4 ft and side slopes of 1V:4H. The ditch slope is 0.010 ft/ft.

Find: Compute the maximum discharge for which this lining will be stable and the corresponding flow depth.

Solution: From Table 8-3, Kentucky bluegrass has a retardance class of C and, from Table 8-4, the permissible shear stress is:

$$\tau_p = 1.0 \text{ lb/ft}^2$$

Then, the allowable depth can be determined by assuming $\tau_p = \tau_d$:

$$y = \tau_p/(62.4S) = 1.0/((62.4)(0.01)) = 1.60 \text{ ft}$$

Now, determine the flow area A and hydraulic radius R:

$$A = y(b + zy) = 1.60\ (4.0 + (4)(1.60)) = 16.64 \text{ ft}^2$$

$$P = b + 2y(1 + z^2)^{1/2} = 4.0 + (2)(1.60)(1 + 16)^{1/2} = 17.19 \text{ ft}$$

$$R = A/P = 16.64/17.19 = 0.97 \text{ ft}$$

Finally, determine the Manning's n value from Figure 8-13, and solve for Q from Manning's equation:

From Figure 8-13, n = 0.072 and:

$$Q = (1.486/n)AR^{2/3}S^{1/2}$$

$$Q = (1.486/0.072)(16.64)(0.97)^{2/3}\ (0.01)^{1/2} = 34 \text{ ft}^3/\text{s}$$

(This method is called the maximum discharge method and is useful for determining the stable channel capacity for a variety of different linings for comparison).

Example 8.A-5

Given: A rectangular channel on a slope of 0.001 with a width of 6 ft expands to a width of 10 ft in a straight-walled transition, Z = 0. The design discharge is 300 ft^3/s and Manning's n = 0.02.

Find: Calculate the depth of flow in the upstream 6 ft wide channel if normal depth is the down-stream control.

Solution: (1) Compute the downstream normal depth y_2:

$[(nQ)/(b^{8/3})(S^{1/2})] = [(0.02)(300)]/[(10.0^{8/3})(0.001^{1/2})] = 0.41$

Then, from Figure 8-4 with z = 0: $y_2/b = 0.65$ and $y_2 = 6.50$ ft

and for a rectangular channel, $y_c = ((Q/b)^2/g)^{1/3} = 3.0$ ft SUBCRITICAL

(2) The specific energy downstream is:

$$E_2 = y_2 + Q^2/(2g\,A_2^2) = 6.50 + (300)^2/((2)(32.2)((10)(6.50))^2) = 6.83 \text{ ft}$$

(3) Choose a straight-walled transition with a divergence angle of 12.5° that has an expansion loss coefficient of 0.5 (HEC 14 *(12)*). The length of the transition would be:

$$L = ((10.0 - 6.0)/2)/\tan 12.5° = 8.9 \text{ ft}$$

(4) Check if subcritical flow is possible by assuming critical depth in upstream channel:

$$E_{1c} = y_{1c} + V_{1c}^2/2g, \text{ and } E_1 = E_2 + z_2 - z_1 + h_L$$

where:

$z_2 - z_1$ = 0.001(8.89) = 0.009 . 0

y_{1c} = $((300/6)^2/32.2)^{1/3} = 4.27$ ft

V_{1c} = 300((6.0)(4.27)) = 11.7 ft/s

V_2 = 300/((6.50)(10)) = 4.6 ft/s

h_L = $0.5(11.7^2 - 4.6^2)/((2)(32.2)) = 0.90$ ft

then: $E_{1c} = 4.27 + 11.7^2/((2)(32.2)) = 6.40$ ft

and: $E_1 = 6.83 + 0 + 0.90 = 7.73$ ft

Now, because $E_1 > E_{1c}$, a subcritical solution exists. If this were not the case, the width of 6.0 ft would have to be increased.

(5) Solve the energy equation, Equation 8.10, by trial:

$$z_1 + y_1 + Q^2/(2g\,A_1^2) = z_2 + E_2 + h_L$$

$$y_1 + (y_1 + (1\text{-}0.5)(300)^2/((2)(32.2)(6.0^2)(y_1^2))$$

$$= \quad 6.83 - 0.5(300)^2/((2)(32.2)((10)(6.50))^2)$$

where: $z_1 - z_2 = 0$
$h_L = 0.5\ (Q^2/(2g\,A_1^2) - Q^2/(2g\,A_2^2))$
$E_2 = 6.66$ ft
$A_2 = (10)(6.50) = 65.0$ ft^2

with the result $y_1 = 6.13$ ft and $V_1 = 8.2$ ft/s

(6) Calculate the water surface profile using the Standard-Step Method if boundary resistance losses are of concern.

Example 8.A-6

Given: A rectangular transition contracts from a width of 10 ft to a width of 5 ft. The approach flow rate is 300 ft^3/s with a depth of 1 ft.

Find: Calculate the depth in the contracted section and the angle and length of the contraction so that the transmission of standing waves downstream is minimized.

Solution: (1) Calculate the approach Froude number for a rectangular channel:

$F = V/(gd)^{1/2} = (300/10.0)(1.0))/((32.2)(1.0))^{1/2} = 5.3$ SUPERCRITICAL

(2) Determine the contraction ratio:

$r = b_3/b_1 = 10/5 = 0.5$

(3) Use Figure 8.A-7:

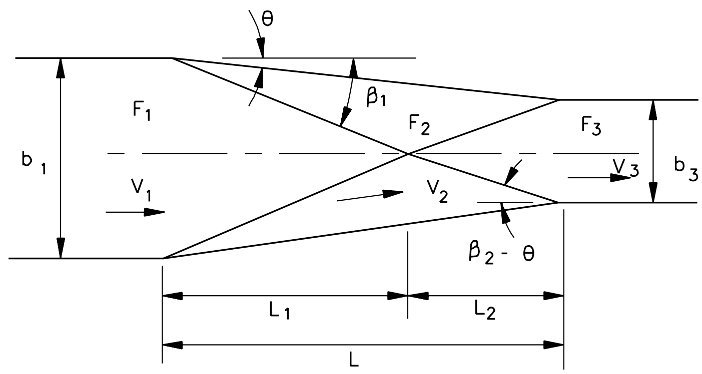
$\theta = 5°$ and $y_3/y_1 = 2.1$ or $y_3 = 2.1$ ft

$F_3/F_1 = r^{-1}\ (y_3/y_1)^{-3/2} = 0.66$, or $F_3 = 3.6$

$L = ((b_3 - b_1)/2)/\tan 5° = 28.6$ ft

(4) This design satisfies the criterion $F_3 > 2$ and also is just to the left of Curve A, which means choking is not possible.

(5) For the complete equations, see HEC 14 *(12)* and Reference *(26)*.



Design of Straight-Wall Contractions in Supercritical Flow

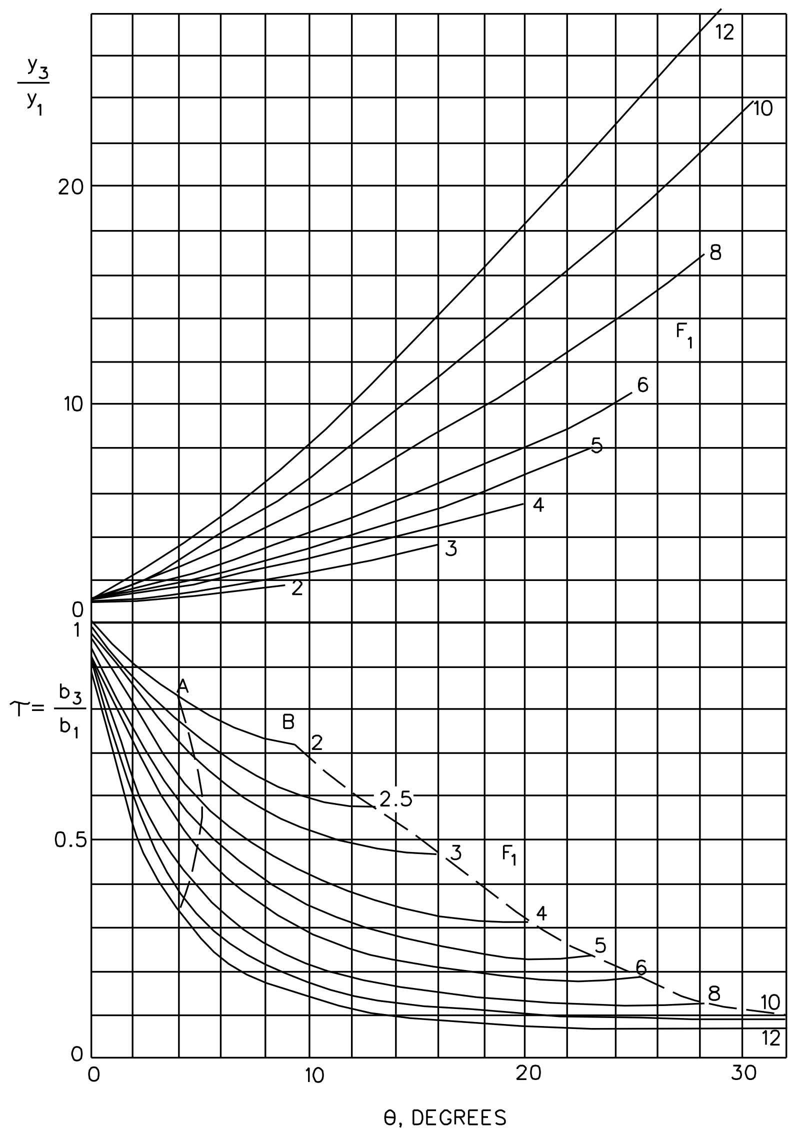


FIGURE 8.A-7 — Contraction Ratio τ and Depth Ratio y_3/y_1 for Supercritical Flow in Contraction of Angle θ (Reference *(26)*)